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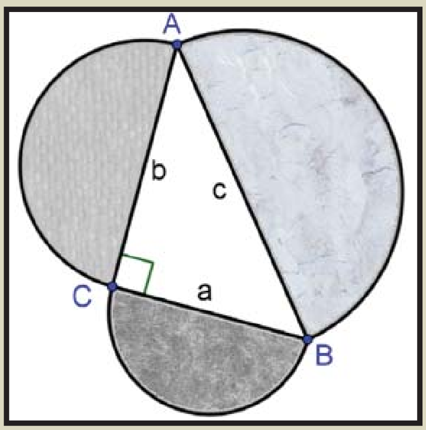
Building an Energetic Community of
Ohio Mathematics Educators



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Ohio Journal of School Mathematics Call for Manuscripts

The *Ohio Journal of School Mathematics* is the journal of the Ohio Council of Teachers of Mathematics. It is intended to be a medium for teachers from elementary to college level to present their ideas and beliefs about the teaching and learning of mathematics. Mathematics educators at all levels are encouraged to submit manuscripts for upcoming issues of the Journal.

Although research studies are not emphasized in the *Journal*, practical application of research implications is appropriate.

GUIDELINES FOR MANUSCRIPTS and ACTIVITIES

Manuscripts should be double-spaced with one-inch margins, 11 point Times New Roman font, and a maximum of 8 pages. References should be listed at the end of the manuscript in APA style. Please include appropriate information such as author, journal or book title, publisher, date, and pages.

Original figures, tables, and graphs should appear embedded in the document – both in the electronic and hard copy forms. Please do not use text boxes, footnotes, or head-notes.

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Submit manuscripts to *Ohio Journal of School Mathematics*, Journal Editors.

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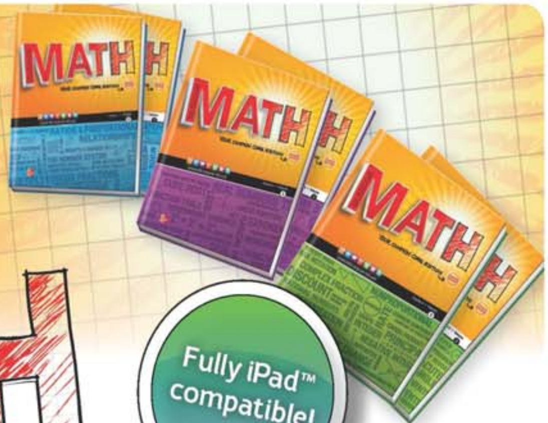

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Ohio Journal of School Mathematics

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Connecting the Art of Navajo Weavings to Secondary Education

Mary Kay Kirchner, Dulaney High School, and
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The Navajo Nation is well-known for its exceptional artistry with respect to the weaving of rugs, blankets, and other textiles. This article will discuss the culture of the Navajo, their weavings, and how this art form can be used to teach and extend mathematics concepts in secondary education. The patterns within the Navajo weavings will be used to illustrate examples of the four isometries and the seven frieze groups. These patterns will also be used to determine the fundamental region, as well as to study the fractal concept of iteration and its impact on area and perimeter.

Introduction

The Navajos, or Diné, as they call themselves, have a philosophy that values beauty and harmony, which can be seen in their weavings (Iverson, 2002). Understanding the geometry behind Navajo weavings requires learning more about the culture, history, and values of the Navajo Nation. For example, "a splendid Navajo rug represents …their worldview [of beauty and harmony within their universe] in an appropriately beautiful and lasting form." (McManis & Jeffries, 2009, p.9). This beauty and harmony, common to all Navajo weavings, is exemplified in the raised outline rug shown in Figure 1.

Bringing the art of Navajo weavings to the secondary curriculum offers educators an opportunity to connect mathematics, art, literature, and history. Cultural concepts can be explored, while noting the history of the Navajo Nation and how weaving became an intricate part of their economy and art. The fictional accounts of the Navajo Nation

Photograph courtesy of National Museum the American Indian, Smithsonian Institution [Catalog Number: 25/4375]

Fig 1 Larry Yazzie, Diné (Navajo) detail of Raised Outline Rug (1994)

Police can educate students on the cultural values and traditions of the Navajo Nation (Hillerman, 1990; Thurlo & Thurlo, 2006). Students can also study the unbreakable code of the Navajo Code Talkers during World War II (Kawano, Gorman, & Frank, 1990).

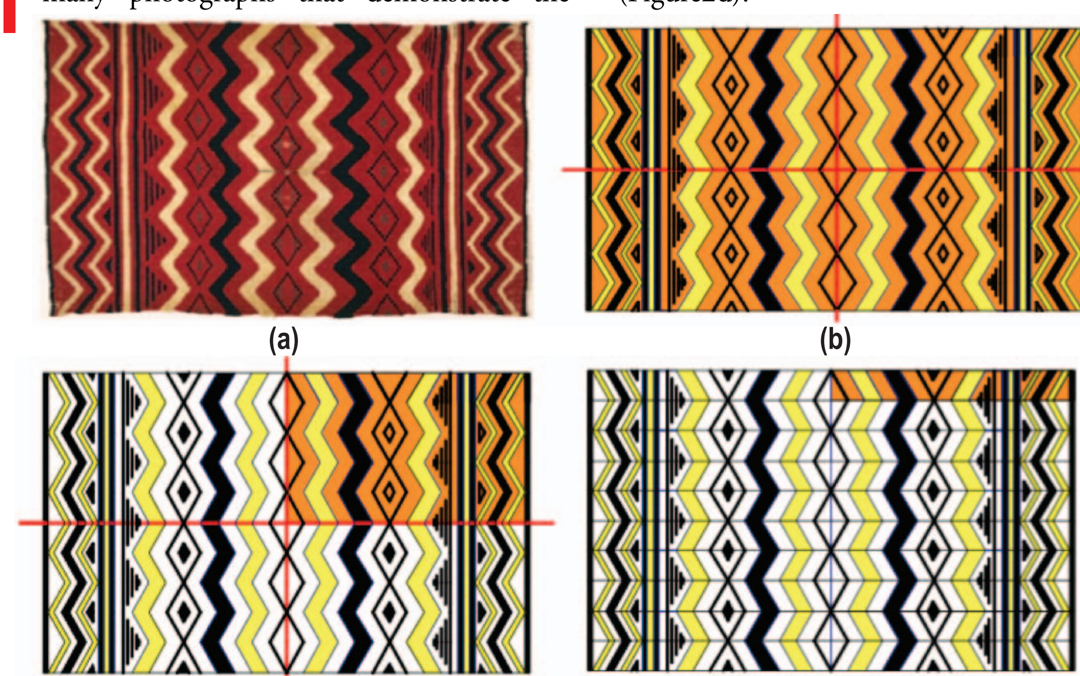
The Geometry of Navajo Weavings

Students can explore the geometry of Navajo weavings, using information in a transformational geometry unit and searching on the Internet for Navajo weavings. A transformational geometry unit typically includes a study of the four isometries: translations, rotations, reflections, and glide reflections. Students may identify isometries used to generate patterns in Navajo weavings.

Students can explore the geometry of Navajo weavings, using information in a transformational geometry unit

The National Museum of the American Indian (NMAI) of the Smithsonian Institution is a great place to start. For students living within driving distance of Washington, D.C., a visit to the NMAI is highly recommended. If an actual visit is not possible, a visit to their website (Smithsonian Institution 2010) provides students with many photographs that demonstrate the geometry, the symmetry, and the beauty of Navajo weavings.

Figures 1, 2, and 3 are Navajo weavings from the 1820s to the 1990s. It is interesting to note the changes in colors and styles, over time. All three weavings may be used to explore and extend the students' knowledge of transformational geometry. Students could study the weaving pattern of the Navajo Poncho in Figure 2a and determine the smallest region possible that can be used to generate the entire pattern, which is often referred to as the "fundamental region" (Washburn & Crowe, 1988, p. 53). Many students immediately notice horizontal and vertical lines of symmetry (Figure 2b). They might decide that 1/2 or 1/4 of the weaving is the smallest region possible to generate the entire pattern (Figure 2c). The pattern can be printed onto paper or copied into Geometer's Sketchpad, allowing students to further explore this concept, by sketching lines directly onto the pattern and searching for the fundamental region. Ultimately, the students will discover that the fundamental region is only 1/20 of the entire weaving (Figure2d).



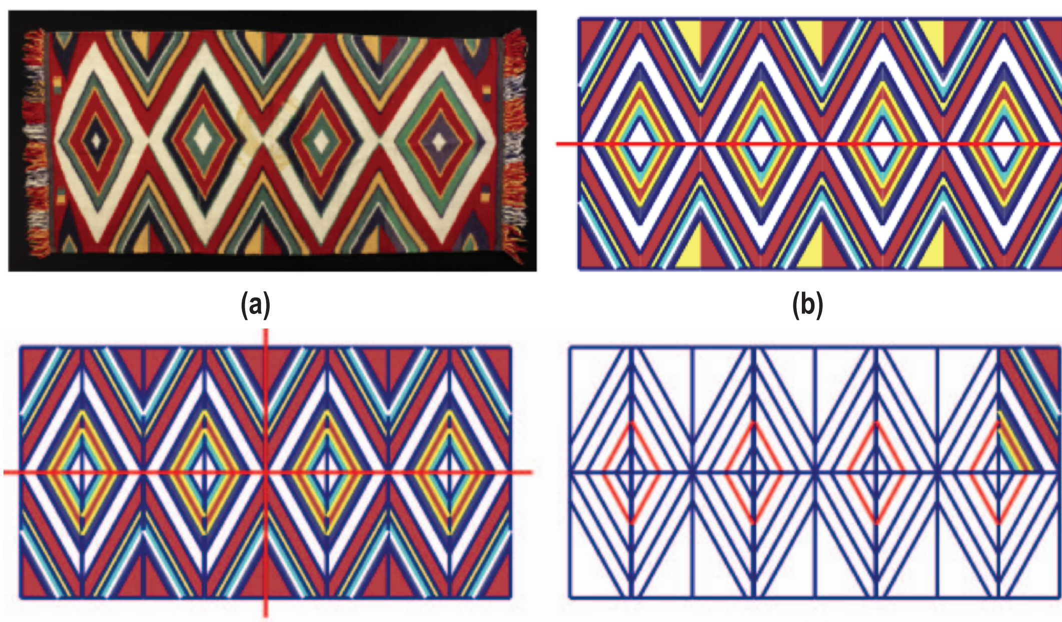
(a) (b) (c) (d)

Photograph (a) courtesy of National Museum of the American Indian, Smithsonian Institution [Catalog Number: 9/1912].

Fig 2 Diné (Navajo) detail of poncho (1825-1860)

The Navajo Double Saddle Blanket in Figure 3a provides another opportunity for students to further explore the concept of the fundamental region. Following a cursory study, students who are asked to find the fundamental region and lines of reflection may quickly decide by using a horizontal line, that the fundamental region is 1/2 of the weaving (Figure 3b). Some students will insist, however, that through the use of an added vertical line of reflection, the fundamental region is actually 1/4 of the weaving. This response will be incorrect, because of the yellow and red triangles at the top and bottom of the weaving.

In response to these findings, the teacher could then ask the students what changes could be made to add a vertical line of reflection. The answer is change the interior color of the three triangles at the top and the bottom of the weaving, from yellow to red or from red to yellow (Figure 3c). This will add the vertical line of reflection and in fact changes the fundamental region from 1/2 of the weaving to 1/16 (Figure 3d). Regardless of the number of transformations used, the simplicity of line, color, and symmetry, turns the textile into a beautiful weaving that has a sense of balance and harmony, reflecting the predominant values in Navajo culture.



(a) (b) (c) (d)

Fig 3 Diné (Navajo) detail of Double Saddle Blanket (circa 1880)

Photograph (a) courtesy of National Museum of the American Indian, Smithsonian Institution [Catalog Number:15/9869].

Using the Seven Frieze Groups to Create Patterns

Students can use their knowledge of the four isometries to discover the seven frieze groups and create new patterns. This portion may be done using graph paper and colored pencils. Too often, we rely heavily on technology, forgetting the discovery power of pencil and paper, but of course technology can be a great tool for mathematical exploration. Many students love the thrill of creating designs using geometry software, such as the Geometer's Sketchpad (Sarhangi, 2009). Regardless of the methodology, students will enjoy creating geometric designs, inspired by their research of Navajo textiles, their knowledge of the four isometries, and a study of the seven frieze groups.

students will enjoy creating geometric designs, inspired by their research of Navajo textiles, their knowledge of the four isometries, and a study of the seven frieze groups.

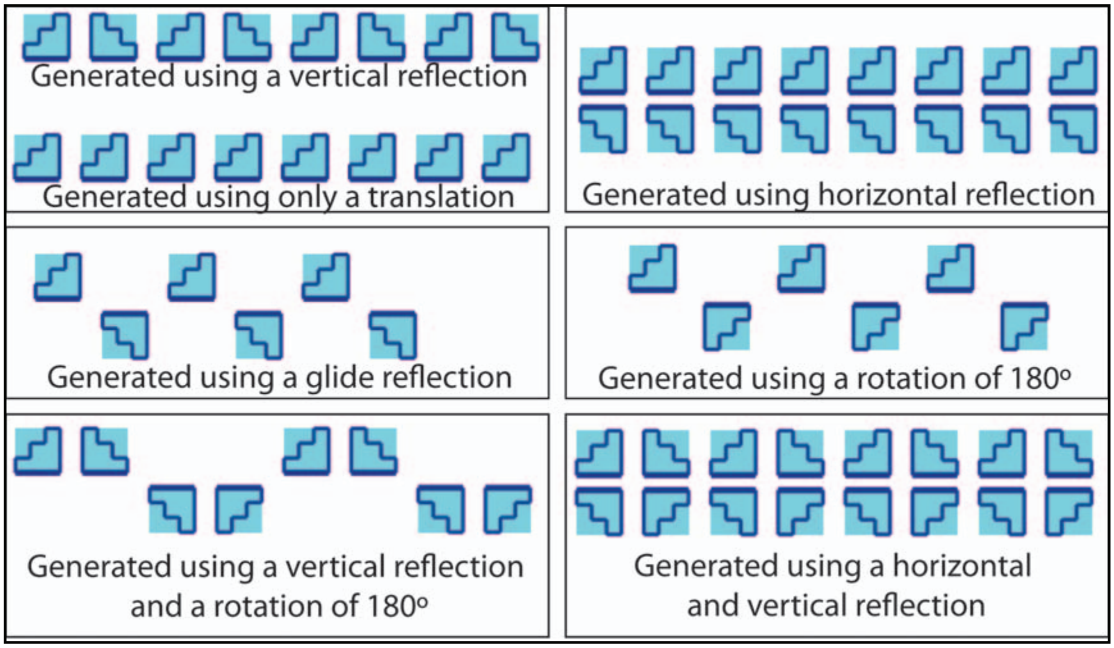


Fig 4 The seven frieze groups

The geometric patterns from Navajo weavings can be used to explore fractal geometry concepts

While most high school geometry students should be proficient at identifying the four isometries, they are probably unfamiliar with the seven frieze groups. "The number of geometrically different patterns for friezes is infinite. Nevertheless, based on their symmetries, all can be classified into seven groups" (Sarhangi, 2000, p. 199). The raised outline rug shown in Figure 1 can be used as a motivational tool, introducing students to the concept of frieze groups. Students could be asked to study the rug and find the three friezes (i.e., the frieze on the left, generated by a translation of a feather, the frieze on the right, produced by a vertical reflection, and a third frieze in the middle, on a diagonal) gracefully balanced between the two right triangles. Students should also notice that this Navajo rug has a sense of balance, achieved by the use of rotating a large, right triangle, but changing the objects in the interior of the triangle.

In an exploration activity, the teacher could guide the students, using either technology or graph paper, to discover the seven freize groups. Washburn and Crowe (1988, p. 83) developed a flow chart with a series of questions that are used to identify a particular frieze group. These questions could be modified so that the students discover the frieze groups for themselves, by taking an existing polygon and applying vertical and horizontal reflections, glide reflections, 180° rotations, and translations along a line. Figure 4 shows the possible outcomes.

Using Fractal Geometry Concepts to Study Area and Perimeter

The geometric patterns from Navajo weavings can be used to explore fractal geometry concepts, by using a sketch from a section of a traditional Navajo rug. The students could be asked to produce a series of iterations from the sketch of the section, measure each new stage and compare the areas, thus affording the opportunity to study series and sequences in a very concrete way.

Students could explore a wide variety of Navajo weaving styles through the website of the Hubbell Trading Post, a National Historic Site of the National Park Service (2010). For students traveling near Ganado, Arizona, the Hubbell Trading Post is well worth the visit.

For those unable to actually visit this historic site, a virtual visit on their website, will provide excellent photographs, demonstrating a wide variety of regional weaving styles. The website also can provide a student with a wealth of information regarding the past and the present of the Navajo nation.

Figure 6 is an adaptation of a pattern, from a section of the Ganado style rug, shown in Figure 5. With each iteration, the shape is growing based on a fractal geometry algorithm, and demonstrates an example of the fractal property of self-similarity, where an object is similar to itself, but in miniature (Sarhangi, 2009).

Suppose that the first shape (on the left) in Figure 6 is comprised of two triangles of 1 square unit each, then the first step has an area of 2(1) square units. After the second step, the area is now 2(1 + 3) square units, with a pattern forming, so that the subsequent iterations are 2(1 + 3 + 5), then 2 (1 + 3 + 5 + 7) or 2, 8, 18, 32, ... as shown in Figure 7.

Photograph courtesy of Hubbell Trading Post National Historic Site

Fig 5 Detail of a round rug with Ganado style

With each iteration, the shape is growing based on a fractal geometry algorithm, and demonstrates an example of the fractal property of self-similarity

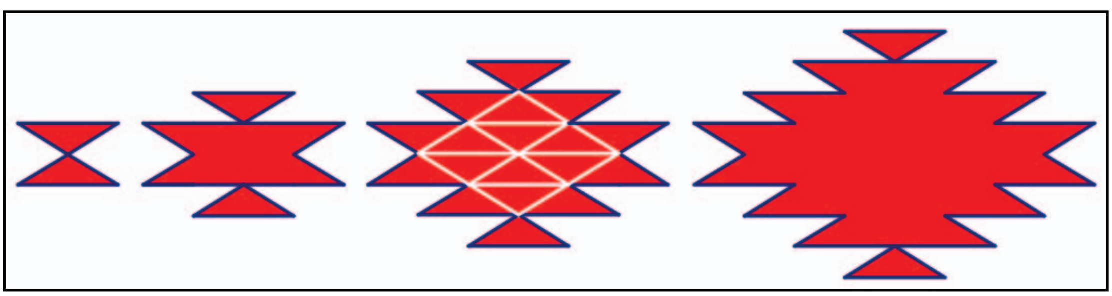


Fig 6 Adaptation of a pattern from the Ganado style rug

First Step, A_1	Second Step, A_2	Third Step, A_3		nth step, A_n
$2(1)$	$2(1+3)$	$2(1+3+5)$	$\rightarrow$	$2(1+3+...+(2n-1))$
2	8	18	$\rightarrow$	$?$

Fig 7 The area of the growing pattern

Using Figure 7 and the formula for the sum of the first *n* terms of an arithmetic progression, students will discover that $Area = 2(1+3+\cdots+(2n-1)) = 2\frac{n[(2n-1)+1]}{2} = 2n^2$.

We can also use Figure 6 to find another sequence based upon perimeter. Using one of the isosceles triangles, if we let the base equal a units and one of the legs equal b units, then the perimeter in the first step is $2a+4b$ units. The second step would have a perimeter of $6a+8b$

units, with the third step having a perimeter of $10a + 12b$ units, and ultimately, a pattern emerges, as shown in Figure 8. Developing the formula for the numerical coefficients in the A_n step is a great opportunity for students to use their knowledge of arithmetic sequences.

First Step, A_1	Second Step, A_2	Third Step, A_3		nth step, A_n
$2a + 4b$	$6a + 8b$	$10a + 12b$	$\rightarrow$	$(4n - 2)a + (4n)b$

Fig 8 The perimeter of the growing pattern

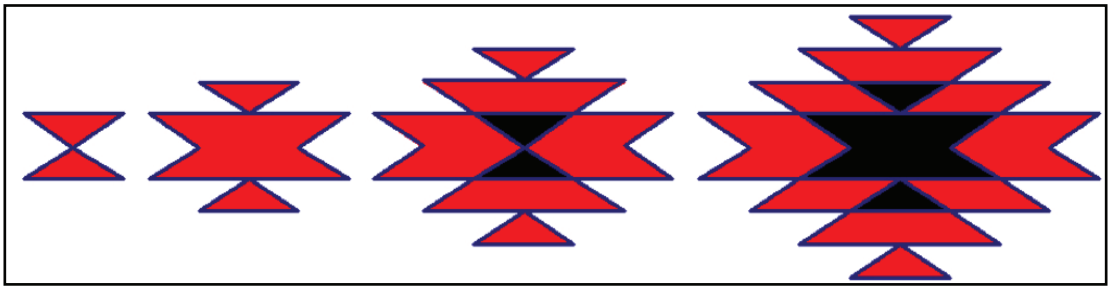


Fig 9 New pattern produced by a color change

By introducing an internal change of design, this same pattern can be analyzed in a different way to produce a different sequence. Referring back to the actual rug as shown in Figure 5, we see that there is a pattern embedded within the main pattern, demonstrating again the property of fractal geometry. Analyzing the red areas, in the stages shown in Figure 9, a related but new sequence emerges as follows. First step: $2(1)$, second step: $2(1 + 3)$, third step: $2(1 + 3 + 5) - 2(1)$. fourth step: $2(1 + 3 + 5 + 7) - 2(1 + 3)$ and continue so that the sequence becomes: $2n^2 – 2(n - 2)^2$, for $n \geq 2$ or more simply as $2(1), 2(4), 2(8), \ldots, 2(4n - 4), \ldots$ or $2, 8, 16, \ldots, 8(n - 1), \ldots$ for $n \geq 2$ as shown in Figure 10.

First Step, A_1	Second Step, A_2	Third Step, A_3		nth step, A_n
$2(1)$	$2(1 + 3)$	$2(1 + 3 + 5) - 2(1)$	$\rightarrow$	$2n^2 - 2(n - 2)^2$
$2(1)$	$2(4)$	$2(8)$	$\rightarrow$	$2(4n - 4)$
2	8	16	$\rightarrow$	$8(n - 1)$

Fig 10 The red area of the growing pattern, with the color change

Conclusion

Navajo weavings in rugs and textiles offer an exciting opportunity in secondary education for students to understand abstract geometric concepts by using concrete hands-on approaches. Navajo weavings provide an inspiration for our students to recognize the beauty of mathematics as well foster a better understanding of the Navajo Nation.

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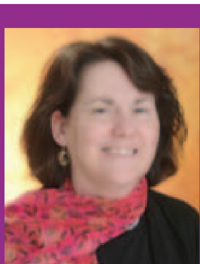

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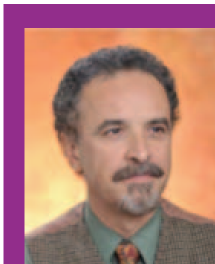

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Wisdom of the Navajo

"You can't wake a person who is pretending to be asleep."

- Navajo proverb (in Kundtz, D. (2009). *Awakened Mind: One-Minute Wake Up Calls to a Bold and Mindful Life.* San Francisco, CA: Cornari Press, p. 7)

The Navajo Nation

"The Navajo of the Southwestern United States are the largest single federally recognized tribe of the United States of America. As of The Navajo Nation has over 300,000 enrolled tribal members."

Donovan, B. (July 7, 2011) "Census: Navajo enrollment tops 300,000." *Navajo Times.* Available on-line at http://navajotimes.com/news/2011/0711/070711census.php

The Relational Mind

"The human brain stores factual knowledge about the world in a relational manner. That is, an item is stored in relation to other items, and its meaning is derived from the items to which it is associated."
Buonomano, D. (2011). *Brain bugs: How the brain's flaws shape our lives, 23*. W. W. Norton & Company, NY.